

演習：行列演算

使用行列

$$A = \begin{pmatrix} 1 & 2 \\ 4 & -3 \\ -2 & 5 \end{pmatrix} \quad B = \begin{pmatrix} 3 & 5 \\ 0 & 1 \\ 4 & -1 \end{pmatrix} \quad C = \begin{pmatrix} 2 & -2 \\ 1 & 4 \\ 0 & 2 \end{pmatrix}$$

$$D = \begin{pmatrix} 4 & 1 & -8 \\ -2 & -1 & 5 \end{pmatrix} \quad E = \begin{pmatrix} 0 & -2 & 3 \\ 4 & 1 & 1 \end{pmatrix}$$

$$F = \begin{pmatrix} 3 & 0 & 1 \\ -1 & 4 & -2 \\ 5 & 1 & 2 \end{pmatrix} \quad G = \begin{pmatrix} 1 & -3 & 7 \\ 1 & 0 & 3 \\ 2 & 4 & 2 \end{pmatrix}$$

$$\mathbf{x} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} \quad \mathbf{y} = \begin{pmatrix} 2 \\ 6 \end{pmatrix} \quad \mathbf{z} = \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix}$$

$$p = 4 \quad q = 5$$

最適化問題を行列で表す1

▶ 最適化問題

▶ 例題: 週末に子供と遊ぶの定式化

$$\begin{aligned} \max. \quad & 4x_1 + 3x_2 \\ \text{s. t.} \quad & x_1 + x_2 \leq 5 \\ & 4x_1 + 2x_2 \leq 16 \\ & x_1, x_2 \geq 0 \end{aligned}$$

$$\begin{aligned} (4 \quad 3) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} &= 4x_1 + 3x_2 \\ \begin{pmatrix} 1 & 1 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} &= \begin{pmatrix} x_1 + x_2 \\ 4x_1 + 2x_2 \end{pmatrix} \end{aligned}$$

▶ 行列で表記した最適化問題

$$\begin{aligned} \max. \quad & (4 \quad 3) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \\ \text{s. t.} \quad & \begin{pmatrix} 1 & 1 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \leq \begin{pmatrix} 5 \\ 16 \end{pmatrix} \\ & \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \geq \begin{pmatrix} 0 \\ 0 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} A &= \begin{pmatrix} 1 & 1 \\ 4 & 2 \end{pmatrix}, \\ c &= \begin{pmatrix} 5 \\ 16 \end{pmatrix}, b = \begin{pmatrix} 5 \\ 16 \end{pmatrix} \text{ と置けば} \\ x &= \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \mathbf{0} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ (と記号で表せば)} \end{aligned}$$

▶ さらに簡略化

$$\begin{aligned} \max. \quad & c^T x \\ \text{s. t.} \quad & Ax \leq b \\ & x \geq 0 \end{aligned}$$

最適化問題を行列で表す2

▶ 最適化問題(例2)

$$\begin{aligned} \max. \quad & 5x_1 + 2x_2 + 3x_3 \\ \text{s. t.} \quad & x_1 + x_2 - 4x_3 \leq 7 \\ & 3x_1 - 2x_2 + x_3 \leq 11 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

$$\begin{aligned} \max. \quad & (5 \quad 2 \quad 3) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \\ \text{s. t.} \quad & \begin{pmatrix} 1 & 1 & -4 \\ 3 & -2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \leq \begin{pmatrix} 7 \\ 11 \end{pmatrix} \\ & \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \geq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} A &= \begin{pmatrix} 1 & 1 & -4 \\ 3 & -2 & 1 \end{pmatrix}, \\ c &= \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}, \quad b = \begin{pmatrix} 7 \\ 11 \end{pmatrix} \\ x &= \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad \mathbf{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \end{aligned}$$

と置けば
(と記号で表せば)

$$\begin{aligned} \max. \quad & c^T x \\ \text{s. t.} \quad & Ax \leq b \\ & x \geq \mathbf{0} \end{aligned}$$