Districting Problem in Japan: An IP-Approach for Graph Partitioning Problem

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The Diet, the parliament of Japan, is a bicameral legislature, with the lower house called the House of Representatives and the upper house the House of Councilors. The House of Representatives has 480 members and the House of Councilors has 252. The election system for the House of Representatives consists of two voting methods. Of the 480 seats, 300 are elected according to the single-seat constituency system, in which one candidate is elected for each of the 300 districts of the country. And the proportional representation system is applied to the remaining 180 seats. We focus on the single-seat constituency system for the lower house.

In Japan, the 300 districting plan is decided by the independent administrative committee, which is chosen by the Government. The current decision process is as follows. Japan has 47 prefectures. First, the committee gives one seat to each prefecture unconditionally. Then, remained 253 seats are apportioned to 47 prefectures by the Largest Remainders Method (LRM) (i.e., the Hamilton's method in US). Insomuch that, for instance, Tokyo prefecture has 1+24 seats. This method is referred as 1+LRM. Next, the committee decides the redistricting plan for each prefecture. Therefore this problem is split up into two parts, the apportionment problem and the redistricting problem. Concretely, the disparity of population size between the largest districts in 300 and the smallest one is 2.064, that is, the ratio is more than 2.

It is objectionable from the principle of "one man, one vote". So, almost all newspapers criticized the plan proposed by the committee. In particular, several articles pointed out the "1+" apportioned part, but threw no light on the LRM and the districting itself. One reason is that it's hard to solve the redistricting problem exactly. The optimal districts could be solved at most 5 districts in Japan. But it is pointed out the necessity to obtain the optimal districts, for example as an index of gerrymander (Sakaguchi and Wada (2000)).

The districting problem can be modeled as the optimization problem. There are several constraints for districts: the districts should be contiguous, the districts should have nearly equal populations, and so on. In the United States, there are several researches for the redistricting problem, but the problem is different from that of Japan. In Japan, the objective is to minimize the population disparity, and the cities shouldn't be divided into smaller parts. However, it is important that the districts should be geographically compact in US (e.g., Mehrotra, Johnson, and Nemhauser(1998)). Thus the redistricting problem in Japan must have been considered separate issues.

We formulate this problem as to two types {0,1}-integer programs; set partition-type and graph partition-type, and then obtained the optimal 300 districts with some ideas for the first time in Japan. Consequently, the population

disparity in the value of one vote is 1.977.

Next, we look at the factors affecting the disparity in the value of votes: the "1+" rule part, the apportionment part, and the decision process itself. If the "1+" rule is removed, that is, 300 seats are only apportioned by the LRM, the disparity ratio is up to 2.032.

We also show that the limit of reduction in the disparity ratio is 1 to 1.750 under the present election law. Specifically, under the "1+" rule, we change in the allocation of seats to prefectures by the several methods; the Largest Divisor (d'Hondt method, Jefferson's method), the Smallest Divisor (Adams's method), the Arithmetic Mean Divisor (Saint-Lagüe method, Webster's method), the Geometric Mean Divisor (Huntington's method, Hill's method), and the Harmonic Mean Divisor (Dean's method). Then we obtained the disparity ratio, respectively.

Furthermore, even if the principle of population proportional apportionment is sacrificed, the ratio is reduced only to 1 to 1.722.

The electoral reform plan to reduce or expand the number of seats is occasionally proposed. However, our mathematical quantitative analysis also shows that the change in the number of seats doesn't work well enough to reduce the vote-value disparity. We derive the disparity ratios with 300 ± 20 members and by several apportioned methods. The minimum limit is 1.747.